$$H[p] = -\int_{\mathcal{X}} \underbrace{p}\, \log p \, dx$$

$\mathcal{X} = \mathrm{supp}(p)$.

$$\langle \phi \rangle_{\underbrace{p}} = \int_{\mathcal{X}} \phi(x)\, p(x)\, dx$$

$$\langle \phi, p \rangle = \int_{\mathcal{X}} \phi(x) p(x)\, dx$$

$x_1, \dots, x_n$ $\quad \hat{\mu}_\alpha = \frac{1}{N} \sum_{n=1}^{N} \phi_\alpha(x_n)$

Find $p \longrightarrow E_p \phi_\alpha = \int_{\mathcal{X}} p \, \phi_\alpha(x)\, dx = \hat{\mu}_\alpha$

$\max H[p]$

$\alpha = 1, \dots, K$

$$\hat{\mu} = E_p \phi = E_p \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_K \end{bmatrix}$$

Tsallias Entropy

$$\max_{\lambda} \frac{1}{N}\sum_{n=1}^{N} \log p(x_n;\lambda)$$

$Ex=\mu$, $Cov=\Sigma$ $\longrightarrow$ $\lambda_1=\Sigma^{-1}\mu$, $\lambda_2=\Sigma^{-1}$

x, xx^T

$$p(x;\lambda)=\exp\{\langle\lambda,\phi(x)\rangle - A(\lambda)\}, \quad A(\lambda)=\log\int\exp(\langle\lambda,\phi(x)\rangle)$$

$$\log p(x;\lambda)=\langle\lambda,\phi(x)\rangle - A(\lambda)$$

$$\frac{1}{N}\sum_{n=1}^{N}\langle\lambda,\phi(x_n)\rangle - A(\lambda) = \langle\lambda, \underbrace{\frac{1}{N}\sum_{n=1}^{N}\phi(x_n)}\rangle - A(\lambda)$$

$$\nabla_\lambda = 0 \Rightarrow \boxed{\underbrace{\frac{1}{N}\sum_{n=1}^{N}\phi(x_n)}_{\hat{\mu}} = \nabla A(\lambda)}$$

$$\text{III} \quad A(\lambda) = \log \int \exp(\langle \lambda, \phi(x) \rangle)\, dx. \qquad \phi = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix} \quad \mathrm{vec}(\Sigma)$$

$$D_\lambda A(\lambda) = \frac{1}{\int \exp(\langle \lambda, \phi(x) \rangle)\, dx} \int \underbrace{D_\lambda \exp(\underline{\langle \lambda, \phi(x) \rangle})}\, dx$$

$$= \frac{1}{\int \exp(\langle \lambda, \phi(x) \rangle)\, dx} \int \exp(\langle \lambda, \phi(x) \rangle)\, \phi(x)\, dx =$$

$$\underbrace{\frac{1}{\int \exp(\langle \lambda, \phi(x) \rangle)\, dx} \exp(\langle \lambda, \phi(x) \rangle)}_{\text{"}p(x;\lambda)}$$

$$= \int p(x;\lambda)\, \phi(x)\, dx = \langle \phi \rangle_p .$$

$$p(x;\lambda) = \exp(\langle \lambda, \phi(x) \rangle \underbrace{- A(\lambda)}) = \frac{\exp(\langle \lambda, \phi(x) \rangle)}{\int \exp(\langle \lambda, \phi(x) \rangle)\, dx}$$

$$\exp(-A(\lambda)) = \int \exp(\langle \lambda, \phi(x) \rangle)\, dx$$