

$$F(\mu, C) = \langle \log \phi(\omega^T h) \rangle_{q(\omega; \mu, C)} \quad \log \phi := \text{concave}$$



$$F(\theta \mu_1 + (1-\theta) \mu_2, \theta C_1 + (1-\theta) C_2) \geq \theta F(\mu_1, C_1) + (1-\theta) F(\mu_2, C_2)$$

$$\left\{ \begin{aligned} \omega &= \theta \mu_1 + (1-\theta) \mu_2 \\ &+ [\theta C_1 + (1-\theta) C_2]^T z \end{aligned} \right. \quad z \sim \mathcal{N}(z | 0, \Sigma)$$

$$\int \mathcal{N}(z | 0, \Sigma) \log \phi [\theta h^T (\mu_1 + C_1^T z) + (1-\theta) h^T (\mu_2 + C_2^T z)] dz$$

$$\geq \theta \int \mathcal{N}(z | 0, \Sigma) \log \phi (h^T (\mu_1 + C_1^T z)) dz + (1-\theta) \int \mathcal{N}(z | 0, \Sigma) \times$$

$$\times \log \phi (h^T (\mu_2 + C_2^T z)) dz$$

$$F(\dots) \geq \theta F(\mu_1, C_1) + (1-\theta) F(\mu_2, C_2)$$

$$| q_t = q_{t-1} + \alpha \epsilon (h_t - q_{t-1}) \quad \left[\begin{array}{l} KL(q'|p) \approx KL(q|p) \\ + \langle \alpha, h \rangle \end{array} \right] \quad \log(1+x) \approx x + O(x^2)$$

$$KL[q + \alpha(h-q) | p] = \int (q + \alpha(h-q)) \log \left(\frac{q + \alpha(h-q)}{p} \right) d\omega =$$

$$= \int (q + \alpha(h-q)) \log \frac{q}{p} \left(1 + \frac{\alpha}{q} (h-q) \right) d\omega =$$

$$= \int (q + \alpha(h-q)) \left(\log \left(\frac{q}{p} \right) + \frac{\alpha(h-q)}{q} + O\left(\alpha \left| \frac{h-q}{q} \right| \right) \right) d\omega =$$

$$= \int \left[\underbrace{q \log \frac{q}{p}}_{KL[q|p]} + \underbrace{\alpha(h-q)}_{=0} + \underbrace{\alpha(h-q) \log \frac{q}{p}}_{\leftarrow} + O(\alpha) \right] d\omega$$

$$\alpha \int (h-q) d\omega = \alpha(1-1) = 0$$

$$KL[q'|p] = KL[q|p] + \alpha \langle h-q, \log \frac{q}{p} \rangle \quad \int (h-q) \log \frac{q}{p} d\omega$$

$$\|u\|_2^2 = \int u^2 \, d\theta$$