$$\underbrace{\log p_\theta(x)}_{\times 1} = \int \log p_\theta(x)\, q(z)\, dz = \{ p_\theta(x)\, p(z|x) = p_\theta(x,z) \} =$$

$$= \int \log \frac{p_\theta(x,z) \times q(z)}{p(z|x) \times q(z)}\, q(z)\, dz = \int q(z) \log \frac{p_\theta(x,z)}{q(z)}\, dz + \int q(z) \log \frac{q(z)}{p(z|x)}\, dz$$

$$\Rightarrow \log p_\theta(x) = \underbrace{\langle \log p_\theta(x,z) \rangle_{q(z)} + H[q]}_{\geq \ \text{Lower-Bound}} + \underbrace{D_{KL}[q(z) \,||\, p(z|x)]}_{\geq 0}$$

$$\underline{q = p(z|x)} \Rightarrow$$

$$\theta = \{\alpha, \beta, \gamma\} \qquad p_0 = \alpha\,[x=1] + (1-\alpha)\,[x=2]$$
$$p_1 = \beta\,[x=2] + (1-\beta)\,[x=3]$$

$$\left[\; p(x_n) = \gamma\, p_0(x_n) + (1-\gamma)\, p_1(x_n) \;\right]$$

$$p(x_n \mid z_n) = \underbrace{p_{z_n}}_{0 \qquad 1}(x_n)$$

$$\int p(x_n \mid z_n)\, p(z_n)\, dz = p(x_n)$$

$$\underbrace{p(z_n = 1)}_{\gamma}\, p_1(x_n) + \underbrace{p(z_n = 0)}_{1-\gamma}\, p_0(x_n) = p(x_n)$$

		1	2	3
p_0	γ	α	$1-\alpha$	0
p_1	$1-\gamma$	0	β	$1-\beta$

$$p(z_n=1 \mid x_n=1) = \frac{p(z_n=1, x_n=1)}{p(x_n=1)} = \frac{p(x_n=1 \mid z_n=1)\,p(z_n=1)}{p(x_n=1 \mid z_n=1)\,p(z_n=1) + p(x_n=1 \mid z_n=0)\,p(z_n=0)}$$

$$= 0.$$

$$\log p(x_n|z_n=k;\theta) = \langle \phi(x_n), \lambda_k \rangle - F(\lambda_k), \quad F(\lambda_k) = \log \int \exp(\langle \phi(x), \lambda_k \rangle)\, dx \qquad p(z_n=k) = \pi_k$$

$$\sum_{n=1}^{N} \langle \log p(x_n, z_n|\theta) \rangle_q = \sum_{n=1}^{N} \langle \log p(x_n|z_n,\theta) \rangle_q + \langle \log p(z_n) \rangle_q$$

$$\overset{(1)}{\langle \log p(x_n|z_n,\theta) \rangle_q} + \overset{(2)}{\langle \log p(z_n) \rangle_q}$$

$$(1) = \sum_{k=1}^{K} q_{nk} [\langle \phi(x_n), \lambda_k \rangle - F(\lambda_k)] = \langle \phi(x_n), \sum_{k=1}^{K} q_{nk} \lambda_k \rangle - \sum_{k=1}^{K} q_{nk} F(\lambda_k)$$

$$(2) = \langle \log p(z_n) \rangle_q = \sum_{k=1}^{K} \log \pi_k \cdot q_{nk}$$

$$\sum_{n=1}^{N} (1)+(2) = \sum_{n=1}^{N} \Big[\langle \phi(x_n), \sum_{k=1}^{K} q_{nk} \lambda_k \rangle - \sum_{k=1}^{K} q_{nk} F(\lambda_k) + \sum_{k=1}^{K} q_{nk} \cdot \log \pi_k \Big] =$$

$$= \sum_{n=1}^{N} \langle \phi(x_n), \sum_{k=1}^{K} q_{nk} \lambda_k \rangle - \sum_{k=1}^{K} F(\lambda_k) \underline{\sum_{n=1}^{N} q_{nk}} + \sum_{k=1}^{K} \log \pi_k \underline{\sum_{n=1}^{N} q_{nk}}$$

$$= \sum_{k=1}^{K} \langle \sum_{n=1}^{N} q_{nk} \phi(x_n), \lambda_k \rangle + \Big(\sum_{n=1}^{N} q_{nk}\Big) \Big[\log \pi_k - F(\lambda_k) \Big]$$

$$\mathcal{F}^{\pi} = \sum_{k=1}^{K} \left(\sum_{n=1}^{N} q_{nk}\right)\left[\log \pi_k\right] + \lambda\left(\sum_{k=1}^{K}\pi_k - 1\right)$$

$$\frac{1}{\pi_k}\left(\sum_{n=1}^{N} q_{nk}\right) = -\lambda \quad \Rightarrow \quad \pi_k = -\frac{1}{\lambda}\sum_{n=1}^{N} q_{nk}$$

$$\sum_{k=1}^{K} \pi_k = -\frac{1}{\lambda}\underbrace{\sum_{k=1}^{K}\sum_{n=1}^{N} q_{nk}} = 1 \quad \Rightarrow \quad \lambda = -N$$

$$\sum_{n=1}^{N} \underbrace{\sum_{k=1}^{K} q_{nk}}_{1} = N$$

$$\pi_k = \frac{1}{N}\sum_{n=1}^{N} q_{nk}$$

$$\mathcal{J} = \sum_{k=1}^{K} \left\langle \sum_{n=1}^{N} q_{nk}\, \phi(x_n), \lambda_k \right\rangle + \left(\sum_{n=1}^{N} q_{nk}\right) \left[\log \pi_k - F(\lambda_k) \right]$$

$$\nabla_{\lambda_k} \mathcal{J} : \sum_{n=1}^{N} q_{nk}\, \phi(x_n) - \left(\sum_{n=1}^{N} q_{nk}\right) \nabla F(\lambda_k)$$

$$\sum_{n=1}^{N} q_{nk}\, \phi(x_n) = \sum_{n=1}^{N} q_{nk}\, \nabla F(\lambda_k)$$

$$\sum_{n=1}^{N} \frac{q_{nk}}{\sum_{n=1}^{N} q_{nk}} \cdot \phi(x_n) = \nabla F(\lambda_k) = \mathbb{E}_{p(x|\lambda_k)} \phi(x)$$

(2) $x \sim p(x;\lambda)$

$$p(x;\lambda,\pi) = \sum_{k=1}^{K} \pi_k \, \mathcal{N}(x \mid \underbrace{\lambda}_{\mu \quad \sigma})$$

$\hat{p}(x)$

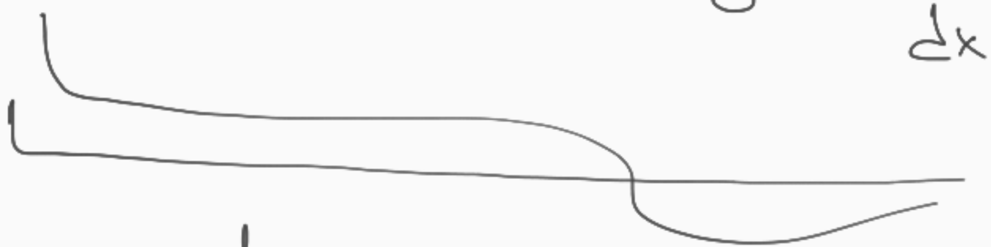
$$D_{KL}(q(x) \,\|\, \hat{p}(x))$$

$$= \int \underbrace{q(x)} \log \frac{q}{\hat{p}(x)} \, dx$$

$$\sum_{k=1}^{K} \pi_k \, \mathcal{N}(x \mid \lambda_k)$$

$q(x \mid z)$

$$\sum_{k=1}^{K} \int \pi_k \, \mathcal{N}(x;\lambda_k) \times \log \hat{p}(x) \, dx$$



$\mathbb{E}_z \int q(x \mid z)$ ..