$$\log p(X,\mu,\lambda) = \left[\sum_{n=1}^{N} \frac{1}{2}\log\lambda - \frac{\lambda}{2}(\mu-x_n)^2\right] + \frac{1}{2}\log(\beta_0\lambda) - \frac{\beta_0\lambda}{2}(\mu-m_0)^2 + (a_0-1)\log\lambda - b_0\lambda$$

$$\log q(\lambda) \propto \frac{N}{2}\log\lambda - \frac{\lambda}{2}\sum_{n=1}^{N}\langle(\mu-x_n)^2\rangle_{q(\mu)} + \frac{1}{2}\log(\lambda) - \frac{\beta_0\lambda}{2}\langle(\mu-m_0)^2\rangle_{q(\mu)} + (a_0-1)\log\lambda - b_0\lambda = \log\lambda\left(\frac{N}{2}+\frac{1}{2}+a_0-1\right) - \lambda\left(\frac{1}{2}\sum_{n=1}^{N}\langle(\mu-x_n)^2\rangle_{q(\mu)} + \frac{\beta_0}{2}\langle(\mu-m_0)^2\rangle_{q(\mu)} + b_0\right)$$

$$\Rightarrow \boxed{q(\lambda) = \mathcal{G}\left(\frac{N}{2}+\frac{1}{2}+a_0,\ \frac{1}{2}\sum_{n=1}^{N}\langle(\mu-x_n)^2\rangle_{q(\mu)} + \frac{\beta_0}{2}\langle(\mu-m_0)^2\rangle_{q(\mu)} + b_0\right)}$$

$$\log q(\mu) = -\frac{\langle\lambda\rangle_{q(\lambda)}}{2}\sum_{n=1}^{N}(\mu-x_n)^2 - \frac{\beta_0\langle\lambda\rangle_{q(\lambda)}}{2}(\mu-m_0)^2$$

$$\nabla_\mu \log q(\mu) = \langle\lambda\rangle\sum_{n=1}^{N}(\mu-x_n) + \beta_0\langle\lambda\rangle(\mu-m_0) = 0$$

$$(\beta_0+N)\mu - \sum_{n=1}^{N}x_n - \beta_0 m_0 = 0 \Rightarrow \mu = \frac{\sum_{n=1}^{N}x_n + \beta_0 m_0}{\beta_0+N}$$

$$\lambda^{-1} = \beta_0+N$$

$$q(\mu) = \mathcal{N}(\ ,\ ).$$

$k = 1, \dots, K$

$$p(x_n \mid z_n) = \mathcal{N}(x_n \mid \mu_{z_n}, (\Lambda_{z_n})^{-1}), \quad z_n = 1, \dots, K.$$

$$\dim z_n = K$$

$$p(x_n \mid z_n) = \prod_{k=1}^{K} \left[\mathcal{N}(x_n \mid \mu_k, (\Lambda_k)^{-1}) \right]^{z_{nk}}. \qquad \sum_{k=1}^{K} z_{nk} = 1, \quad \forall k \; z_{nk} \in \{0, 1\}.$$

$$r_{nk} = p(z_{nk} = 1)$$

$$\log q(\pi) \overset{+}{\propto} \langle \log p(X, Z, \pi, \Lambda, \mu) \rangle_{q(Z, \Lambda, \mu)} =$$

$$\overset{+}{=} \left\langle \sum_{n,k} z_{nk} \log \pi_k + \sum_{k} (a_0 - 1) \log \pi_k \right\rangle_{q(Z)} = \sum_{n,k} \underbrace{\langle z_{nk} \rangle_{q(Z)}} \log \pi_k + \dots$$

- $$p(z_j \mid z_{-j}, x) = h(z_j) \exp\{ \langle \phi(z_j), \eta(z_{-j}, x) \rangle - F(\eta(z_{-j}, x)) \}$$

$$\log p(z_j \mid z_{-j}, x) = \log h(z_j) + \langle \phi(z_j), \eta(z_{-j}, x) \rangle - \cancel{F(\eta(z_{-j}, x))}$$

$$\langle \log p(z_j \mid z_{-j}, x) \rangle_{q(z_{-j})} = \log h(z_j) + \langle \phi(z_j), \mathbb{E}_{-j} \eta(z_{-j}, x) \rangle$$

$$q(z_j) = h(z_j) \exp\{ \langle \phi(z_j), \mathbb{E}_{-j} \eta(z_{-j}, x) \rangle - F(\mathbb{E}_{-j} \eta(z_{-j}, x)) \}.$$