$$P_\mu = (x_k-\mu)(x_k^2+\lambda) = \underbrace{(x_k-\mu)}_{a}\{[(x_k-\mu)+\mu]^2+\lambda) = a((a+\mu)^2+\lambda) \quad ; \quad Ea=0.$$

$$E\,a([a+\mu]^2+\lambda) = E\,a(a^2+2\mu a+\mu^2+\lambda) = E\,\underbrace{a^3}_{0} + 2\mu a^2 + \underbrace{\mu^2 a}_{0} + \underbrace{\lambda a}_{0} = 2\mu.$$

$$E[a([a+\mu]^2+\lambda)]^2 = E\left[a^2(a^2+2\mu a+\mu^2+\lambda)^2\right] = E\left[a^2\left((a+\mu^2)^2+(2\mu a+\lambda)^2 + 2(\mu a+\lambda)(a^2+\mu^2\right)\right]$$

$$= E\{a^6 + a^4(2\mu^2+2\lambda+2\mu^2) + a^2\underbrace{(\mu^4+\lambda^2+2\lambda\mu^2)}_{(\mu^2+\lambda)^2} + 0\cdot a^0.$$

$$Ea^6 = 15$$
$$Ea^4 = 3 \quad \Rightarrow \quad \boxed{15+3(6\mu^2+2\lambda)+(\mu^2+\lambda)^2}$$
$$Ea^2 = 1$$

$$\|V = 15+3(6\mu^2+2\lambda)+(\mu^2+\lambda)^2 - 4\mu^2 = V(\lambda^*)$$

$$\min_\lambda 6\lambda + (\lambda^2+2\lambda\mu^2) \Rightarrow \lambda^* = -(\mu^2+3).$$

$$15+3\begin{pmatrix}6\mu^2\\-2\mu^2\\-6\end{pmatrix}$$
$$-4\mu^2$$
$$+\begin{pmatrix}\mu^2-\mu^2\\+3\end{pmatrix}$$

$$= 15+3(4\mu^2-6)-4\mu^2+3$$

$$\min_{\mu} \langle (\varepsilon+\mu)^2 \rangle_{\mathcal{N}(\varepsilon;0,1)}$$

$$\nabla_{\mu} \int (\varepsilon+\mu)^2 \mathcal{N}(\varepsilon;0,1)\,d\varepsilon = \int 2(\varepsilon+\mu)\mathcal{N}(\varepsilon;0,1)\,d\varepsilon =$$

$$= 2\Big[\overbrace{\int \varepsilon\, \mathcal{N}(\varepsilon;0,1)\,d\varepsilon}^{=0.} + \mu \int \mathcal{N}(\varepsilon;0,1)\,d\varepsilon\Big]$$

$$= 2\mu.$$

$$\approx 2(\varepsilon_k+\mu), \quad \varepsilon_k \sim \mathcal{N}(\varepsilon|0,1)$$

$$\mathbb{V}\, 2(\varepsilon_k+\mu) =$$

$$= 4.$$

$$\min_{\mu} \langle \varphi(\varepsilon + \mu) \rangle_{\mathcal{N}(\varepsilon|0,1)}$$

$$\varphi = G(\varepsilon + \mu)$$

$$\partial_{\mu} \int \varphi(\varepsilon + \mu) \mathcal{N}(\varepsilon|0,1)\, d\varepsilon = \int \partial_{\mu}\varphi(\varepsilon + \mu)\, \mathcal{N}(\varepsilon|0,1)\, d\varepsilon$$

$$\varepsilon_k \sim \mathcal{N}(\varepsilon|0,1)$$

$$\approx \partial_{\mu} \varphi(\varepsilon_k + \mu) \;.$$

$x \sim \mathcal{N}(x; \mu, I_d)$

1. $\langle f_i(x) \rangle = 0$

2. $a \le D f_i \le b$

$$\max_{\mu} \left\langle \sum_{i=1}^{d} f_i(x_i) \right\rangle_{p(x;\mu)}$$

$$\nabla_{\mu_j} \left\langle \sum_{i=1}^{d} f_i(x_i) \right\rangle_{p(x;\mu)} = \left\langle (x_j - \mu_j)\left(\sum_{i=1}^{d} f_i(x_i) + \lambda\right) \right\rangle_{p(x;\mu)}$$

$$\approx (\check{x}_j - \mu_j)\left(\sum_{i=1}^{d} f_i(\check{x}_i) + \lambda\right) = \varepsilon$$

$$V\left[\sum_{i=1}^{d} (\check{x}_j - \mu_j)\left(f_i(\check{x}_i) + \frac{\lambda}{d}\right)\right] = \overbrace{V\left[\sum_{i \ne j} (\check{x}_j - \mu_j)\left(f_i(\check{x}_i) + \frac{\lambda}{d}\right)\right]}^{(1)} + \boxed{V\left[(\check{x}_j - \mu_j)\left(f_j(\check{x}_j) + \frac{\lambda}{d}\right)\right]}_{\ge 0}$$

$j \perp i$

$$(1) \sum_{i \ne j} V\left[\underbrace{(\check{x}_j - \mu_j)}_{E(x_j - \mu_j) = 0}\underbrace{\left(f_i(\tilde{x}_i) + \frac{\lambda}{d}\right)}_{\frac{\lambda}{d}}\right] = \underbrace{\sum_{i \ne j}}_{(d-1)} \underbrace{V(\check{x}_j - \mu_j)}_{1} \underbrace{V\left(f_i(\tilde{x}_i) + \frac{\lambda}{d}\right)}_{\ge a} \ge a(d-1).$$

$$\partial_\mu := \langle (x_j - \mu_j)\left(\sum_i f(x_i) + \lambda\right)\rangle =$$

$$= \underbrace{\langle (x_j - \mu_j)(f(x_j) + \lambda)\rangle}_{} + \underbrace{\langle (x_j - \mu_j)\left(\sum_{i \neq j} f(x_i) + \lambda\right)\rangle}_{}$$

$$\mathbb{E} = \mathbb{E}_{-j}\mathbb{E}_j$$

$$\left(\sum_{i \neq j} f(x_i) + \lambda\right)\underbrace{\mathbb{E}_j (x_j - \mu_j)}_{0} = 0$$

$$p(x;\lambda) = \exp(\langle t(x), \lambda\rangle - F(\lambda))$$

$$p(\lambda;\tau,n_0) = H(\tau,n_0)\exp(\langle\lambda,\tau\rangle - n_0 F(\lambda))$$

$$p(\lambda|x) \propto \gamma\, \underbrace{p(x;\lambda)\, p(\lambda;\tau,n_0)}\, + (1-\gamma)\, p(x;\lambda)\, p(\lambda;\check{\tau},\check{n}_0)$$

$$= H(\tau,n_0)\,\underbrace{\exp(\langle\lambda,\tau+t(x)\rangle - (n_0+1)F(\lambda))}\,. \quad H(\tau+t(x), n_0+1)$$

$$p(\lambda|x) \propto \gamma\,\frac{H(\tau,n_0)}{H(\tau+t(x), n_0+1)}\, p(\lambda;\tau+t(x), n_0+1) + (1-\gamma)\frac{H(\check{\tau},\check{n}_0)}{H(\check{\tau}+t(x), \check{n}_0+1)}\, p(\lambda;\check{\tau}+t(x), \check{n}_0+1).$$

$$\int(\ldots)\,d\lambda = \gamma\,\frac{H(\tau,n_0)}{H(\tau+t(x), n_0+1)} + (1-\gamma)\,\frac{H(\check{\tau},\check{n}_0)}{H(\check{\tau}+t(x), \check{n}_0+1)}.$$